Wave Form Design For QAM-FBMC System
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Abstract - Multiple filter-banks enable QAM transmission and reception in filter-bank multi-carrier (FBMC) systems. Due to its spectral efficiency and well-localized spectrum, a quadrature amplitude modulation based filter-bank multi-carrier (QAM-FBMC) is considered as one of the candidate for 5G mobile communications. In this paper, we propose a novel waveform design for the QAM-FBMC system considering time dispersion parameter. The utilization of the proposed filter with the real number filter coefficients enables simpler implement than the conventional multiple prototype filters with complex coefficients. Simulation results show that the proposed QAM-FBMC systems have superior spectral efficiency as compared to CP-OFDM. Moreover, a new designed filter shows the improvement in the performance compared with the conventional QAM-FBMC systems on multipath fading channel.

Keywords— QAM, FBMC, multi carrier, spectral efficiency

I. INTRODUCTION

In conventional wireless communication standards such as LTE and Wi-Fi, the cyclic prefix orthogonal frequency division multiplexing (CP-OFDM) has been considered as a waveform in the physical layer. However, the rectangular pulse shape of the CP-OFDM raises an out-of-band emission problem in the frequency domain, and the time domain redundancy is required due to the CP. Because of these disadvantages, the CP-OFDM is not appropriate for the requirements of 5G communications such as high spectral efficiency and flexible spectrum for various environments. To overcome the obstacles, the various multi-carrier schemes called as new waveforms have been studied [1]

The filter-bank multi-carrier (FBMC) has been considered as a good candidate technology for enabling 5G communications [2]. The FBMC utilizes the designed pulse shaping filter on each subcarrier to enhance the spectrum localization and provides flexibility in the waveform design. To avoid an interference problem by the overlapped pulse shaping filter, offset-quadrature amplitude modulation (OQAM)-FBMC is introduced [3]. However, the OQAM scheme causes an intrinsic interference in the practical complex channels, and it is difficult to implement the conventional pilot design, channel estimation, and MIMO technologies.

To avoid the intrinsic interference problem of the OQAM-FBMC, a QAM-FBMC system based on multiple prototype filters is proposed in It suggests the filter coefficients set to satisfy the generalized Nyquist condition and the fast fall-off rate condition properly. Although this research shows a fine trade-off relationship between the orthogonality and spectrum localization of the filter, the interferences from near-orthogonal filters still exists. To mitigate the residual interference, an enhanced QAM-FBMC scheme based on decision feedback equalization (DFE) are proposed in [5].

In this paper, we suggest a simple waveform design for the QAM-FBMC considering the time domain localization based on single prototype filter. The filter coefficients are composed of real numbers, so that the proposed filter is much simpler to implement. Furthermore, the filter is more adequate for the practical channel than the conventional filter. From simulation results, the proposed filter shows comparable bit error performance as in the CP-OFDM and superiority of the spectrum localization. Also it shows better bit error performance than the conventional QAM-FBMC system in multi-path fading channels.

II. SYSTEM MODEL IN QAM-FBMC SYSTEMS
A. Transmit Signal Model

The discrete-time transmit signal x[n] of the QAM-FBMC system is represented as the sum of complex QAM data symbols dm[k] on the m-th subcarrier in the k-th symbol as follows:

\[ x[n] = \sum_{k=-\infty}^{\infty} \sum_{m=0}^{M-1} dm[k] pm[n - kM] \quad (1) \]

where M is the number of subcarriers. The time domain (TD) filter coefficients pm[n] on the m-th subcarrier are well-designed pulse shaping filter, and its frequency domain (FD) filter coefficients are given with up-sampling between subcarriers.

Fig. 1. Overlap & sum structure in the QAM-FBMC system (L = 4)
In QAM-FBMC systems, the time domain filters \( p_n \) are generated by applying corresponding frequency shift of the prototype filter \( p_0[n] = q[n] \). Therefore, the filters are represented as follows:

\[
P_m[n] = q[n]e^{j2\pi mn/M} \tag{2}
\]

With the representation in (2), we can rewrite the transmit signal as follows:

\[
x[k] = \sum_{n=-\infty}^{\infty} p_0[n - kM] \sum_{m=0}^{M-1} d_m[k] e^{j2\pi mn/M} \tag{3}
\]

which can be simply implemented using M-point inverse fast Fourier transform (IFFT), and TD filtering based on poly phase network as shown in Fig. 1. The TD filter coefficients \( pm[n] \) is defined on \( \{n|0 \leq n < N\} \), and \( \sum_{n=0}^{N-1} |p_m[n]|^2 \). The FBMC symbol duration \( T \) is defined by \( N = LM \), and \( L \) is an up-sampling factor. Typically, the QAM-FBMC symbol duration is \( L \times M \) longer than \( M \), so that the QAM-FBMC system is organized by overlap and sum structure as depicted in Fig. 2. As a result, the neighboring QAM-FBMC symbols partially overlap each other with \( M \) samples shifting. Moreover, we can tell that the CP-less OFDM is the special case of (3) with \( L = 1 \) and \( p_0[n] = \frac{1}{\sqrt{M}} \).\( \forall n \).

### B. Receive Signal Model

For block processing of the system, we define the transmitted data symbol vector of length \( M \) in the \( k \)-th QAM-FBMC symbol as \( d[k] \) with the \( m \)-th element \( d_m[k] \). Then we can rewrite the \( k \)-th transmitted symbol vector \( x[k] \) as follows:

\[
k[k] = W_N^H P_f[k] \tag{4}
\]

where \( P_f \) is the \( N \times M \) frequency domain filter coefficient matrix in which the \( m \)-th column is given by \( N \)-point discrete Fourier transform (DFT) of the corresponding shifted filter of the prototype filter \( p_0[n] \), and \( W_N \) is the \( N \)-point DFT matrix with \( (m,n) \)-th entries

\[
(W_N)_{m,n} = \frac{1}{\sqrt{N}} e^{-j2\pi mn/N}.
\]

Using equation (4) and overlap and sum structure of the transmitted symbols, the 0-th received symbol vector of length \( N \) is represented as follows:

\[
x[k] = \sum_{k=-L}^{L-1} T[k] H[k] x[k] + w[0] \tag{5}
\]

where the additive white Gaussian noise (AWGN) vector \( w[0] \) with zero mean and variance \( \sigma^2 \) is identically independent Gaussian distributed. The channel matrix \( H[k] \) is a Toeplitz matrix with size \( (N + M) \times N \), and each column of the matrix is given by the circular shift of the channel impulse response as follows:

\[
[H]_{k,l} = \text{circshift} \{[h_0 \ldots h_{L-1} \text{O}_{N+M+L-1}]^T, n-1 \} \tag{6}
\]

where \( L_c \) is the length of the time domain channel taps. The shift-and-slice matrix \( T[k] \) with size \( N \times (N + M) \) is defined as [6].

\[
\begin{bmatrix}
0 & 1_{N\times M+L} \\
0 & 0 \\
1_L & 0 \\
0 & 0 \\
1_{N\times M+L} & 0 \\
\end{bmatrix}, \quad k < 0 \\
\begin{bmatrix}
1_L & 0 \\
0 & 0 \\
\end{bmatrix}, \quad k = 0 \\
\begin{bmatrix}
0 & 1_{N\times M+L} \\
0 & 0 \\
1_{N\times M+L} & 0 \\
\end{bmatrix}, \quad k > 0
\tag{7}
\]

which is efficient to denote the interferences to the 0-th received symbol from neighboring symbols. In equation (5), the summation index from the \(-L+1\) to the \((L-1)\) is interpreted in interferences from adjacent symbols by the QAM-FBMC overlap and sum structure as shown in Fig. 2, and the \(-L\) term should be included by the tails of causal channel.

Let the 0-th received QAM-FBMC symbol in frequency domain is \( Y[0] \), which is the \( N \)-point FFT output of (5). If we apply the appropriate channel equalizer \( G_c[0] \), then the estimated data symbol vector of the 0-th symbol can be written as [5]

\[
\hat{x}[0] = P_f^H G_{eq}[0] Y[0] = P_f^H G_{eq}[0] \sum_{k=-L}^{L-1} T[k] H[k] x[k] + w[0], \tag{8}
\]

where \( G_{eq}[0] \) is the up-sampled frequency domain equalizer with the size \( N \times N \), and \( w[0] \) is a noise after equalization and filtering.

### III. FILTER DESIGN FOR QAM-FBMC SYSTEM

According to the Balian-Low theorem, it is difficult to maintain the orthogonality in a multi carrier system with well localized function and Nyquist symbol density at the same time. Therefore, the compromises are considered among the localization, the orthogonality, and the Nyquist rate. The QAM-FBMC filter design should be conducted by relieving the orthogonality condition within acceptable performance loss to enhance spectrum localization property. Because the pulse shaping filters per subcarrier are generated by the prototype filter \( q[n] \), only the design of prototype filter enable to operate the QAM-FBMC systems. In order to determine the elaborate filter coefficients, three parameters are considered in the following.

#### A. Self Signal to Interference Ratio(Self-SIR)

First, we define the overall pulse shape as follows:

\[
g_o[n] = \pm p_{[n]}^* p_{[n]} \tag{9}
\]

which represent the cascade of the i-th transmit pulse shape and j-th receive filter. For the purpose of avoiding inter symbol interference (ISI) and inter-carrier interference (ICI) concurrently, the overall pulse shape (9) should satisfy the generalized Nyquist criterion (GNC) as follows

\[
g_o[nM] = \delta_o \delta_{n-L} \tag{10}
\]

The rectangular pulse shape in the OFDM can assure the condition (10). However, as mentioned above, the perfect orthogonality and well-localized spectrum is incompatible at the same time. Thus, the we should consider self-interference, and the total self-interference

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power at the i-th subcarrier $I_i$ can be calculated by using the overall pulse shape (9) as follows:

$$I_i = \sum_{j} \sum_{k=(L-1)}^{L} |a_{i,j}[kM]|^2 - |a_{i,i}[0]|^2$$

(11)

If we assume that the power of the prototype filter is normalized to unity ($a_{i,j}[0] = 1$), then we can define the self-SIR on the i-th subcarrier [4] as the inverse of the total self-interference power $I_i$.

B. Fall-off rate

First, we give a definition of the fall-off rate as the decreasing rate of the power spectral density (PSD) of the QAM-FBMC transmit signal. Assuming that the sequence of data is uncorrelated, zero-mean, and unit variance, then the discrete time Fourier transform (DTFT) of the pulse shape $q(t)$, $Q(e^{jw})$ represents the PSD of the QAM-FBMC signal. Therefore, the fall-off rate is involved in the decrement of $Q(e^{jw})$. Now, we draw the relation between the FD filter coefficients $Q[k]$ and the fall-off rate of $Q(e^{jw})$ by using the continuity and differentiability of the filter [9]. We assume that the prototype filter $q(t)$ in FD satisfies the conjugate symmetric, i.e., $Q[k]=Q^*[−k]$, and the prototype filter have K number of one-sided non-zero filter taps, now we can derive the continuous-time function of the prototype filter as follows:

$$p_c(t) = \sum_{k=(K-1)}^{K-1} Q[k] e^{j2\pi \frac{t}{\tau}}$$

$$= Q[0] + 2 \sum_{k=1}^{K-1} \Re [Q[k] e^{j2\pi \frac{t}{\tau}}], \quad 0 \leq t \leq NT.$$  

(12)

From the extension of the contents in [9], the condition for fall-off rate is simplified as follows:

$$Q[0] + 2 \sum_{k=1}^{K-1} \Re [Q[k]] = 0,$$ \hspace{1cm} (13a)

$$\sum_{k=1}^{K-1} k^r \Im [Q[k]] = 0,$$ \hspace{1cm} (13b)

$$\sum_{k=1}^{K-1} k^q \Re [Q[k]] = 0,$$ \hspace{1cm} (13c)

where $r$ is an odd positive integer and $q$ is an even positive integer. In the end, the fast fall-off rate is obtained by the numbers of differentiability at the boundary point of TD filter response. To be more exact, if (12) is continuous with the maximum integer number $p$ which satisfies (13b) and (13c), then the fall-off rate of ($w$) would be approximately $|W|-(p+2)$.

C. Time dispersion

To design the sophisticated prototype filter, we should consider the both time and frequency dispersions of the pulse-shape [2]. The frequency dispersion can be adjusted by constraining the one-sided number of non-zero filter taps [4]. Thus, the time dispersion parameter can be defined as follows:

$$\sigma_t = \sqrt{\sum_{n=0}^{N-1} (n-\mu)^2 |p(n)|^2},$$

(14)

where, $\sum_{n=0}^{N-1} |p[n]|^2 = 1$, and $\mu = \sum_{n=0}^{N-1} n |p[n]|^2$. Note that the time dispersion parameter $\sigma_t$ specifies the...
standard deviation of the pulse shaping filter energy in the time domain. If the pulse shaping filter is well-localized in the time domain, the time dispersion of the filter is smaller than otherwise.

Although the QAM-FBMC systems do not use the CP, the pulse shaping filter with small \( \sigma_t \) would be beneficial for multi-path fading channel.

D. Filter design method

The QAM-FBMC filter design problem is identical to the determination of the prototype filter coefficients \( Q[k] \). In order to obtain the proper value, an optimization problem is solved with minimization of the fitness function under the two constraints: fast fall-off rate, small time dispersion as follows: minimize \( I \) subject to

\[
Q[0] + 2 \sum_{k=1}^{K-1} \text{Re}[Q[k]] = 0,
\]

\[
\sum_{k=1}^{K-1} k \cdot \text{Im}[Q[k]] = 0,
\]

\[
\sum_{k=1}^{K-1} k^2 \cdot \text{Re}[Q[k]] = 0,
\]

where the \( \varepsilon \) is the time dispersion parameter. By setting up the maximum number satisfying the fall-off rate condition (13b) and (13c) is 2, we also achieve the \( |W|^{-4} \) fall-off rate of \( Q(w) \). To obtain the small time dispersion, the real filter coefficients in FD is preferred due to the symmetry of pulse shaping filter. Therefore, we achieve the \( |W|^{-5} \) fall-off rate of \( Q(w) \) by satisfying the condition \( \sum_{k=1}^{K-1} k^3 M[Q_k]=0 \), naturally.

<table>
<thead>
<tr>
<th>Filter</th>
<th>Reference [4]</th>
<th>Proposed</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Case 1</td>
<td>Case 2</td>
</tr>
<tr>
<td>( K ) (taps)</td>
<td>4</td>
<td>15</td>
</tr>
<tr>
<td>Self SIR (dB)</td>
<td>10.6</td>
<td>19.4</td>
</tr>
<tr>
<td>Fall-off rate</td>
<td>(</td>
<td>\sigma_t</td>
</tr>
<tr>
<td>( \sigma_t/(N-1) )</td>
<td>0.149</td>
<td>0.197</td>
</tr>
<tr>
<td>Coefficient</td>
<td>Complex</td>
<td>Complex</td>
</tr>
</tbody>
</table>

Although we can configure the up-sampling factor \( L \) variously, we set up the up-sampling factor \( L = 4 \) to compare the performance with the reference filter [4] fairly. In addition as shown in Fig. 3, the filter design is particularly effective when the number of filter taps is \( K = \left( \frac{2m+1}{2} \right) L + 1 \), \( m = 1, 2, 3 \ldots \), where \( [x] \) is the floor operation. By using global optimization algorithm, we propose the filter coefficients set whose number of filter taps is 7, 11 and 15 when up-sampling factor \( L = 4 \) in Table 1.

IV. SIMULATION RESULTS

In order to demonstrate spectrum localization and BER performance of the QAM-FBMC systems, we utilize the filter presented in Table 1. In addition, we set the parameters as \( M = 64 \), \( L = 4 \), \( NCP = 6 \), and QPSK/16QAM modulation. The extended pedestrian A (EPA) model [10] and the zero forcing (ZF) equalizer is considered.

A. Bit error rate (BER) performance

We obtain the bit error rate (BER) performances over AWGN and EPA channels. Fig. 3 shows that the BER performance of the OFDM and the QAM-FBMC systems on AWGN channel. There is some performance degradation in the QAM-FBMC systems compared to the OFDM because of the inherent interference by overlap and sum structure. The smaller self-SIR QAM-FBMC has the worse performance. However, the proposed filter would be suitable for practical channel such as EPA model [10] because time dispersion of the proposed filters is smaller than the filters proposed in [4]. In Fig 4, the performance comparison results are plotted on EPA channel. The filter Case C has better performance than Case 2 despite of low self-SIR, because the interference from channel delay is less vulnerable by the small time dispersion.

![Fig 3. Self-SIR versus normalized number of filter taps.](image)

![Fig 4. Probability of bit error (BER) vs. Es/No.](image)
consider time domain localization. In the simulation, the proposed QAM-FBMC system achieved the comparable BER performance to the CP-OFDM, while the superior spectrum confinement was obtained. Thus, the proposed QAM-FBMC system is expected to be a replacement of the CP-OFDM in various environments.

REFERENCES


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